QP CODE: 24018741

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , APRIL 2024

Second Semester

CORE - ME010202 - ADVANCED TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

F3855757

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let A and B be subsets of a space X and suppose there exist a continuous function f from X to unit interval such that f(x)=0 for all x in A and f(x)=1 for all x in B. Show that there exist disjoint open sets U and V such that A⊆U and B⊆V
2. Let A be a subset of space X and f:A→R be continuous show that two extensions of f to X agrees on closure of A
3. Explain the concept of a wall.
4. Given that each of the coordinate spaces $\{(X_i, \tau_i) : i \in I\}$ is T_1. Prove that their topological product is T_1.
5. A product space is completely regular. Prove that each of its coordinate space is completely regular.
6. Define evaluation function of a family of functions.
7. Show that every continuous real valued function on countably compact spce is bounded and attains its extrema.
8. Is sequential compactness preserved under continuous functions? Justify
9. Define a directed set. Give an example.
10. If a net $S : D \to X$ has $x \in X$ as limit then prove that so does every subnet of S.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let X be a T_2 space and $x \in X$. Let F be a compact subset of X not containing x. Show that there exist disjoint open sets U and V in X such that $x \in U$ and $F \subseteq V$ and hence deduce that a compact subset in a T_2 space is closed

12. Let A be a closed subset of a normal space X and suppose $f : A \to [-1,1]$ is a continuous function. Show that there exist a continuous function $F : X \to [-1,1]$ such that $F(x) = f(x) \forall x \in A$

13. 1) Define the finite product of sets in terms of certain functions whose domain is the index set and using this define the product when the index set is uncountable
    2) What is the cartesian product $\{2\} \times R$ in R^2

14. If X_i is a T_1 space for each $i \in I$. Prove that $\prod_{i \in I} X_i$ is a T_1 space in the product topology

15. Obtain necessary and sufficient condition for the evaluation function of a family of functions to be one-to-one.

16. Prove that if $\mathcal{C}$ is a locally finite family and $\mathcal{D} = \cup_{C \in \mathcal{C}} C$. Then prove that $\overline{\mathcal{D}} = \cup_{C \in \mathcal{C}} \overline{C}$ and that $\{\overline{C} : C \in \mathcal{C}\}$ is locally finite.

17. Show that a space is Hausdroff if limits of all nets in it are unique

18. Prove that the relation homotopy between functions is an equivalence relation

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Show that every regular Second countable space is normal

20. If the product space is nonempty. Prove that each co-ordinate space is embeddable in the product space.

21. (a)Prove that a topological space is completely regular if and only if the family of all continuous real-valued functions on it distinguish points from closed sets
    (b)Let $\{f_i : X \to Y_i | i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Then prove that the corresponding evaluation function $e : X \to \prod_{i \in I} Y_i$ is open when regarded as a function from X onto $e(X)$.

22. When you say that a net is eventually in a subset A of X. Let (D, $\geq$) is a directed set and E an eventual subset of D.

(a) Show that E with restriction of $\geq$ is a directed set.

(b) Prove that a net $S : D \to X$, where X is a topological space, converges to x in X iff the restriction $S/E : E \to X$ converges to x in X

(2×5=10 weightage)